Advanced Math: Notes on Lessons 50-53
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Lesson 50: Trigonometric Equations

Both sin 45° and sin 135° are $\frac{1}{\sqrt{2}}$; what's more, since adding multiples of 360° doesn't change the value of sine, there are an infinite number of sine arguments that produce that answer. Most of the time we don't need all of them, just the ones inside one full rotation. So often an answer will add:
$0° \leq \theta < 360°$

One way to double-check to make sure that you get the right answers is to draw by hand the curve for sin/ cos / tan.

Lesson 51: Common Logarithms and Natural Logarithms

Actually, you already know this (we covered this earlier) - so double this up with another lesson. Basically, logarithms can be of any base, but three are especially common: 10, e (which is 2.71828....), and 2.

"ln x" is just a common shorthand for "$\log_e$ x" (log to base e), and is called the "natural logarithm".

When you use 10 as the base, that's the "common logarithm". Saxon states that "log" without any base subscript is a common logarithm (i.e., that "10" is the default). That is a common but *not universal;* you can use that convention in-class but I suggest *not* depending on it elsewhere. Many people use "lg x" as a shorthand for "logarithm base 10" that is more unambiguous.

The base 2 version is sometimes abbreviated as "ld x" or "lb x". That's not noted in Saxon, I'm just noting it for completeness (in computing, unsurprisingly, base 2 is the most common form).

Calculators often have at least an "ln" button that computes the logarithm base e, and a "log" or "lg" button that computes the logarithm base 10. Try them out!

Lesson 52: The inviolable argument / Arguments in Trigonometric equations

The first part is just a reminder of what you *cannot* do with a function. If you have f(2x)=6, you *cannot* "divide both sides by 2" to produce f(x)=3. Once something is inside a function, you can only expand out the function, or find rules for *that particular function* for moving information in and out of it.

This can make solving equations with trig functions complicated if there is some multiplier of the argument. [Walk through the tan 3θ = 1 example]

Lesson 53: Review of Unit Multipliers / Angular Velocity

Unit multipliers are simply "multiplying by 1"; you usually do this when you're converting to different units. Since 12 inches = 1 foot, multiplying by (12 inch/1foot) is the same logically as multiplying by one, but in fact will also convert the units. You've used these before, this isn't really new.

Angular velocity is probably very new to you, though. But it's very similar to something you already know.

Normal velocity is simply distance (including direction) / time, in other words, the rate that distance is traveled over time. Note that velocity has a direction.

Angular velocity measures the rate of rotation of an object, and it also has a direction.

The linear velocity of a point on a rotating body is:

$$v = r\omega$$

where:

- v = linear velocity (the velocity of the kind that you're already familiar with)
- r = radius (distance from center of rotation)
- ω = angular velocity in radians/time.

Sometimes you get angular velocity in degrees/time, or revolutions/time (a revolution is equal to 360°), so to use this equation you'll first need to convert to radians (2π radians = 360°).