Advanced Math: Notes on Lessons 33-36
David A. Wheeler, 2013-10-22

Lesson 33: Quadrilaterals
Memorize the names and properties; they'll be used in later proofs (e.g., you'll be told ABCD is a rhombus... and then be expected to use the properties of a rhombus).

Rectangle: all angles are right angles. Rhombus: All sides are congruent
So a square is both a rectangle and a rhombus, and anything that's a rectangle and a rhombus is a square.

Lesson 34: Summation, Linear Regression, Decomposing Functions

Summation notation makes it easy to show a sum of many terms. Generally,

$$\sum_{x=low}^{high} f(x) = f(low) + f(low+1) ... + f(high)$$

So for example:

$$\sum_{x=2}^{4} x^2 + 1 = ((2)^2+1) + ((3)^2+1) + ((4)^2+1) = 5 + 10 + 17 = 32$$

The "linear regression" part has you approximate a line using your eyes, and then determine the equation of a line from it. To determine the slope of a line, use *far-apart* points, because tiny measurement mistakes will get amplified if you use points that are close together. Nobody approximates lines using just your eyes in "real life"; normally the lines are calculated using the "least squares" approach. This lesson is mainly to show what "least squares" does; you'll actually use it in lesson 45.

The "decomposing functions" lesson is simply about reversing the composition operator we saw earlier. Remember that (f○g)(x) = f(g(x)). If f(x)=x+2, and g(x)=1/x, then (f○g)(x) =f(g(x)) = f(1/x) = (1/x)+2.

Now imagine that we only got (1/x)+2, and were asked to find f and g where (f○g)(x)=(1/x)+2. How can we do that? Given only that information, there are an infinite number of pairs f,g that solve it. Let's find one nontrivial one for:
(f○g)(x) = (1/x)+2
f(g(x)) = (1/x)+2 ; definition of f○g.

You can solve this "inside out" or "outside in". I tend to do "inside out", so let's look at the items "next to" x... do you see a pattern? Well, looks like the first thing that happens to x everywhere is the reciprocal, so let's set
g(x)=1/x. Now let's solve for f:
f(g(x)) = (1/x)+2
f(g(x)) = g(x) + 2 ; because g(x)=(1/x)

Now we have "g(x)" on both sides of the equation, so we can substitute "x" for "g(x)" on both sides of the equation (yes, you CAN do that):
f(x) = x + 2

Lesson 35: Change in Coordinates
This one is straightforward.

Lesson 36: Angles greater than 360, sums of trig functions, boat-in-river
Angles that differ by multiples of 360 are called "coterminal". Most of the times, coterminal angles can be considered equal (they point the same way!)... in particular, the trig functions will produce the same results. So, you can add/subtract 360 to get "easier" angles. There is one exception, not mentioned in Saxon: sometimes in physics/engineering, they indicate the amount of rotation of something... e.g., 720° means "turned it counterclockwise twice". In 2D positions that difference doesn't matter (the final direction is the same), but in some circumstances (e.g., turning a screw or engine) it *does* make a difference (because there's something else other than the final pointed-to direction that matters).

The "sums of trig functions" is simply practice with trig functions and adding/subtracting radicals. You already know how to do these; the main issue is to make sure you can do these without error.

The boat-in-the-river problems are just more velocity x time = distance problems. The difference is that the "velocity" is actually two things, added together: the speed of the water, plus the still-water speed of the boat (i.e., the speed of the boat relative to the water). Obviously, moving water can speed your progress (compared to the shore position) if you move in the direction of the water... and impede your progress if you move against it. So it's just:
(total_velocity) x time = distance

But since the total velocity is a combination of boat and water speed, it becomes:

(boat_speed + water_speed) x time = distance ; if you're moving *with* the water
(boat_speed - water_speed) x time = distance ; if you're moving *against* the water

The same thing happens when flying in air, but now it's airspeed (not water speed) that matters.

Normally adding or subtracting rates (including speeds) is a *mistake*, but you *do* need to do it when speeds are moving against each other. So when adding speeds, make sure it's this kind of situation.

You'll notice I used the term "velocity" and then switched to "speed" above. The two words have a subtle difference in meaning. "Velocity" includes both the *direction* and *speed,* while speed just contains, well, speed. So a car can have a speed of 60mph, and a velocity of "due North at 60mph". We have these as two different words because often it *matters* what direction something is going.

Do home study test #8 this week (covers lessons 29-32), presumably on Friday. Please review your notes before taking it, and memorize what you need first. *Show your work*, so can give partial credit. Have a parent grade it initially (to identify which ones are right or wrong), and put it in the new mailbox on Sunday morning. I intend to do the final grading, so that I can give partial credit (or full credit if it’s just an equivalent expression).