Advanced Math: Notes on Lessons 29-32
David A. Wheeler, 2008-10-09

Lesson 29: Unit Circle (what's tan) / Large & small fractions / Quadrantal angles
Since sin x = opp/hyp, cos x = adj/hyp, and tan x = opp/adj, then
(sin x / cos x) = (opp/hyp) / (adj/hyp) = ((opp/hyp)*hyp)/adj = opp/adj = tan x. Thus,
tan x = sin x / cos x. There are actually many "trig identities", this is an easy one.

The "limit" information here is really important; it's the basis of calculus. Basically, when variables get closer and closer to some value, expressions using them often converge closer and closer to some other value, and we can express that "other value" as a "limit". So 1+(x/10) gets close to 1 as x gets close to 0. This is a key tool for dealing with division-by-zero; we can't directly divide by zero, but limits sometimes let us get around this restriction.

E.G., what's sin 90°? Strictly speaking, there's no triangle at that angle; if the "opposite" and "hypotenuse" are zero, we might think the answer is 0/0. But if we say "as x gets closer and closer to 90°, what value does sin x approach", then the answer is that "the limit of sin x, as x approaches 90°, is 1"... so that's what the answer is. [Show circle as angle approaches 90°; then show sine wave.]

Lesson 30: Addition of vectors / Overlapping triangles
Adding vectors is simple when the the vectors in rectangular form; add the x's together for the new x, and add the y's together for the new y. E.G.., given vectors:
v1 = (5, -4)
v2 = (2, 3)
The final (resultant) vector v1+v2 = (5,-4)+(2,3) = (5+2, -4+3) = (7,-1).
Vectors are critical for building large structures (bridges, buildings, etc.); you can represent the various forces on it as vectors, add them up, and see what the final force is (too much will rip it apart).

This lesson focuses on when they *aren't* in rectangular form. Sometimes vectors are expressed in polar form, that is, in terms of length and an angle: (r,θ). So, how can you add two vectors where at least one is in polar form, such as (r1,θ1)+(r1,θ2)?

First, there are two special cases that are easy:

- The two angles are equal (θ1=θ2). In this case, the result has the same angle, and you just add the two "r" ("length") values. So (5,45º) + (6,45º) = (11,45º).
- The two angles are opposite of each other (e.g., θ1=θ2±180º). In this case, pick one of the vectors whose angle will be the "final" angle (usually you'd pick the one with the bigger "r" value). Change the other vector by negating its "r" value and changing its angle to be the same as the first one - then add. So (5,45º) + (6,225º) = (-5,225º)+(6,225º)=(1,225º).

In all other cases, you need to convert the vectors to rectangular form. Once you convert it to rectangular form, adding them is easy, just add the matching parts. You can convert polar form (r,θ) to its equivalent rectangular form (x,y) by computing:

$x = r \cos \theta$, $y = r \sin \theta$

So (5,25º)+(7,80º) = (5 cos 25º, 5 sin 25º)+(7 cos 80º, 7 sin 80º)
= (4.532,2.113)+(1.216,6.894)= (4.532+1.216 , 2.113+6.894) = (5.748, 9.007)

Lesson 31: Symmetry / Reflections / Translations
In this lesson, Saxon describes how to take an original function and change it. His description is accurate, but might be confusing, because he doesn't give his "changed" functions different (new) names. Here, I'll talk about the "original" function f, and then I'll discuss creating "new" functions with different names like "g" and "h". Hopefully, by giving these modified functions *different* names, what's going on will be clearer than what's in the book.

The reflection of function f(x) in the y-axis is g(x) = f(-x). So if $f(x) = x^2+8x+6$, its reflection $g(x) = f(-x) = (-x)^2+8(-x)+6 = x^2-8x+6$. If a function's reflection is itself, then it is *symmetric* about the y-axis. E.G., if you have $f(x)= x^2$, then its reflection $g(x) = f(-x) = (-x)^2 = x^2$ which is the same as f(x).

Note: -f(x) is typically *different* from f(-x). f(-x) takes its input (x), negates it, and *then* feeds it to function f, while -f(x) takes its input, feeds it to function f, and then negates the *result* of f.

Similarly, you can move a function to the left or right by adding a constant to x everywhere. A function that has the same shape, but has been "moved" to another place, is called a *translated* function, and the process of doing this is called *translation*. E.G., given some function f(x), the translated function g(x)=f(x+h) has been moved by h spaces to the left; the translated function h(x)=f(x-h) has been moved to the right. E.G., given:

f(x)=2x+2

We can find a new function g(x) that is f(x) moved 3 spaces to the left:

g(x) = f(x+3) = 2(x+3)+2 = 2x+6+2 = 2x+8

Lesson 32: Inverse functions / Four Quadrant Signs / Inverse Trig functions
Every one-to-one function has an inverse function; in fact, that's why one-to-one functions are important. An "inverse" function is a function that "goes the other way".

The sin/cos/tan functions can take arbitrary angles, but then they aren't one-way functions (and can't be inverted). Having an inverse function is very useful... so, to make that possible, we have to create a variant of sin/cos/tan that *is* a one-way function, and then create an inverse of *that*. So the standard convention is arccos returns 0°...180°, Arcsin returns -90°..90°, arctan returns -90°...90°; this convention was chosen so that they *all* cover 0°...90°, and connect continuously for their entire range of values. [Show graphs.]

If we didn't adopt these conventions, we couldn't create inverse functions for sin/cos/tan. We don't have to have single-value functions, but the math to handle arbitrary infinitely-sized sets of values is *much* more complex (trust me, you don't want to do it). In most real-life circumstances, that kind of complexity is completely unnecessary. By adopting a few simple conventions, we make our lives much easier.

Do home study test #7 this week (covers lessons 25-28), presumably on Friday. Please review your notes before taking it, and memorize what you need first. *Show your work*, so can give partial credit. Have a parent grade it initially (to identify which ones are right or wrong), and put it in the new mailbox on Sunday morning. I intend to do the final grading, so that I can give partial credit (or full credit if it's just an equivalent expression).
Note: In problems where it says "do not use a calculator", it's especially important to show your work... so I can see that you didn't!