Advanced Math: Notes on Lessons 13-16
David A .Wheeler, 2007-09-18

Lesson 13: Circles with Secants / Intersecting Secants / etc.
This is a hodgepodge of facts about combining circles and lines/segments. You need to **memorize these** so you can use them. They're pretty straightforward, so I don't plan to spend much class time on them. Circles and lines are both common in the real world, so combinations of them are common too.

For 13.A, "two secants intersect inside a circle", it might be easier to remember that the sum of the two equivalent internal angles (x) equal the sum of the arcs' angle measures. The "divide by 2" in the book is because the two internal angles are equal, i.e., $(x+x)/2 = 2x/2 = x$.

Lesson 14: Sine (Sin), Cosine (Cos), Tangent (Tan), Rectangular/Polar Coordinates
This lesson discusses the sine, cosine, and tangent functions, and should be review. When written in an equation, mathematicians always use their (lower-case) abbreviations: sin, cos, and tan.

These functions (including their inverses) are *incredibly* useful; they let you convert between angles and lengths. They're so common that computing them is built into the Intel/AMD chips used in the majority of desktop/laptop computers.

I'm a believer in "Soh Cah Toa", though as long as you remember these facts, use what works:
sin a = **o**pposite/**h**ypotenuse
cos a = **a**djacent/**h**ypotenuse
tan a = **o**pposite/**a**djacent

(Walk through example 14.1)

The book uses the inverse of these functions, but doesn't clearly explain here that the inverse of these functions have names and "gotcha" issues. The inverse functions are named (respectively) arcsine, arccosine, and arctangent, abbreviated arcsin, arccos, and arctan (or asin, acos, atan). On many calculators, INV SIN computes the arcsine. With *lots* of limitations, arcsin(sin(A)) = A. So if you have an equation of the form:

$\sin A = X$

you can solve for A by doing an "arcsine" on both sides, producing:

$\arcsin(\sin A) = \arcsin X$
$A = \arcsin X$

But the inverse trig functions have lots of limitations. First, angles outside the range 0..360° can't be distinguished from angles measuring 0..360° (try it!). Second, none of these functions can even distinguish between all of those values, e.g., sin 0° = 0, and sin 180° = 0 too... so what should arcsin(0) be? By convention these inverse functions return a *conventional principal angle* value, which may not be exactly the same value you put in. They *are* truly inverses if the original value was between 0° and 90° inclusive. Note: arctan has problems at 90° and 270° (division by 0). Lesson 32 will give more details.

(Walk through example 14.2.)

Beware! The functions sin, cos, and tan all take *angle* measures, and their inverses *produce* angle measures. Most handheld calculators normally measure angles in *degrees* (0..360°), but most programs on desktops/ laptops/ PDAs (including spreadsheet functions) measure angles in *radians* (0..2π), and there are other ways to measure angles too (esp. gradians). Strictly speaking, these different angle measurement systems aren't different "unit systems", but practically speaking, treat them as unit systems and *mark* them (e.g., mark degrees with °). (A common abbreviation for radians is "rad"; in math, if you don't identify the angle measurement system, radians is usually assumed). Be sure you know how to get your calculator to compute these in *degrees*.

Angles are measured from the horizontal (towards the horizon), so angles going "up" are angles of elevation. An "angle of depression" measures the other way, going down (watch out for signs; an angle of depression of 30° is the same as an angle of elevation of -30°). The line of sight, aka slant range, aka range, is the length of the segment connecting directly to the object.

(Walk through) Example 14.4 solves the problem oddly. If it makes sense to you, great. But note that this setup forms a triangle with 26°, opposite length of 3000ft, and unknown hypotenuse, so this also works:
sin 26° = opposite/hypotenuse
sin 26° = 3000 ft./range
0.438371147 = 3000 ft./range
range = 3000 ft / 0.438371147
= 6843.52 ft

Rectangular coordinates are usually given as (x,y). Rectangular/polar coordinate conversion is used widely, including in many videogames. Often videogames will store locations internally as (x,y), because it's easier to store information about a virtual world that way. But if you look in some direction (angle) from "your" specific position, the videogame will need to determine the angles of each other object and how far away they are – resulting in polar/rectangular conversions.

Convert polar (r, θ) to rectangular (x,y): $x = r \cos \theta$, $y = r \sin \theta$. This always works!

Convert rectangular (x,y) to polar (r, θ): $r^2 = x^2 + y^2$, and $\tan \theta = y/x$
But converting this direction is trouble. Besides the division by 0 if x=0, arctan only gives you a *principal* value (typically -90° to 90°), not necessarily what you need. The problem is the individual signs of x and y matter: e.g., (-x, -y) is not the same as (x, y), but -x/-y = x/y. Similarly, (-x, y) is different from (x, -y), but -x/y = x/-y. One way to deal with this problem, as shown in the book, is to set up the problem so that the angle is between 0 and 90° (inclusive), then add the "rest of the angle" after you're done so that you get the conventional angle measure. (In the real world you'd take the arctan, and add 180° as needed to get the correct quadrant with conventional angle measure; most computer systems have an "atan2" function that does this.)

You need to know how to form the 4 variations of a polar coordinate (using negative magnitude, negative angle, or both); see example 14.5. Remember: You can always add or subtract 360° to an angle to get the same angle; adding or subtracting 180° will cause the angle to "reverse direction".

Lesson 15: Assumptions/Proofs
For class, from now on, geometric proofs will be in the 2-column format (statement + reason). Even in non-geometric proofs, it's best to think this way: For every step, know *why* you can make that step.

Lesson 16: Complex fractions / abstract equations / division of polynomials
This lesson should be much easier than lesson 14, and will hopefully counterbalance the time for this

week. 16A and 16B discuss tricks to simplify complicated fractions quickly, and should be fairly clear.

Lesson 16C shows how to check polynomial division. It appears more complicated than it is, and I have a trick to make it faster/easier. Remember that dividend/divisor=quotient, and obviously, if you compute 633/3, you should produce 211. How can you check if 211 is right?

- Well, if you multiply 211 by 3/3, you should produce 633/3. This is the general approach the book suggests; the book suggests taking the quotient, and multiplying it by "divisor/divisor"; you should produce "dividend/divisor". This is quite correct, but a little slow as a check; the problem is that you have to keep copying the "/A" part as you do the work(/3 here).
- A variant of this checking procedure is to multiply 211 by 3, and show that it's 633. More generally, multiply quotient by divisor, and show that it's the dividend. Then you don't have to keep writing the /A part.

Either way works as a check, and are mathematically the same approach. This lesson is about doing the same thing with polynomials; the main issue is that you have to be careful to not "drop" anything.

Also, when multiplying polynomials, if you do it "long hand" with the entire (non-remainder quotient) + (remainder / divisor) it's a *lot* of work. Instead, do the two parts separately (non-remainder and remainder), and then add them. E.G., if you need to calculate and check:
$(x^3 + 2x + 4) / (x-1)$

(Walk through) You should through long division determine that this is:
$= (x^3 + 0x^2 + 2x + 4) / (x-1) = x^2 + x + 3 + 7/(x-1)$
Note that the quotient has two parts: a non-remainder quotient, $x^2 + x + 3$, and the remainder / divisor.

How to check this? Well, multiply the whole thing by the divisor, and you should get the quotient. But if you just naively multiply the entire quotient by the divisor, you can get yourself into extra work:
$x^2 + x + 3 + 7/(x-1)$
$\quad * \quad x - 1$
So instead, since a*(b+c) = a*b + a*c, multiply the divisor by the non-remainder quotient, and separately multiply the divisor by the remainder/divisor (=remainder). Then add them up. This is much simpler, esp. if the divisor is complicated:
$(x-1)(x^2 + x + 3) + (x-1)(7/(x-1))$
$= (x^3 + 0x^2 + 2x - 3) + 7$
$= (x^3 + 2x + 4)$ HOORAY! It's correct !

THIS WEEK:
Work through lessons 13-16 this week ("Monday – Thursday").

Do home study test #3 this week (covers lessons 9-12), presumably on Friday. This will be our first real, graded test. Please review your notes before taking it, and memorize what you need first. Although it primarily covers lessons 9-12, as with all Saxon tests, it'll cover previous material (lessons 1-8) too. *Show your work*; I can't give partial credit without that!

Have a parent grade it initially (to identify which ones are right or wrong), and bring it in to the next class. I intend to do the final grading, so that I can give partial credit (or full credit if it's just an equivalent expression).

The test should only take about an hour, but you can take up to 3 hours if you need it. This is a closed book test. You can use a calculator as long as it's an "ordinary" calculator that only does numerical calculations (no fancy symbolic calculators please). A calculator won't help much on this test; I did the entire test #3 without one. Do questions 19 and 20, but you can simply draw them by hand to show that you understand the definitions of the key terms. Questions 19 and 20 ask you to construct something, but I am not interested in constructions—I just want to see if you understand the definitions of the key terms.

Note: next home study test (#4), which we'll do *next* week, will use trig functions. So for next week's test you *will* need a calculator that has buttons for sin, cos, and tan on it. Any scientific calculator (even the $10 kind) will have these functions.