More Calculus Examples

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Differentials

The "differential" is a new function that tells you the slope, at every point, of some original function.

You can differentiate simple polynomials with terms cx^n by replacing each term with cnx^{n-1} (multiply, then decrement the exponent). This only works as-is when "x" is a simple variable.

Given $f(x)=3x^2 + 4x – 1$, what is its differential $f'(x)$?   Answer: $6x + 4$.

Find $\dfrac{d}{dx}(3x^7+4x^6+22x^3+7)$.   Answer: $21x^6 + 24x^5 + 66x^2$.

Find $\dfrac{d}{dx}(-3x^6+0.5x^4+8x^2+7)$. Answer: $-18x^5 + 2x^3 + 16x$.

Find $\dfrac{d}{dx}(3x^{12}-7x^4-2x-7)$.   Answer: $36x^{11} – 28x^3 – 2$.

Find the slope at x=4 for function $f(x) = 7x^4 – 3x^2 – 2$.

Answer: The differential $f'(x) = 28x^3 – 6x$; we want slope at x=4, $f'(4)= 28(4)^3 – 6(4) = 1768$.

Indefinite Integrals (Anti-derivatives)

The indefinite integral (anti-derivative) is just the reverse of the differential.

Find the indefinite integral for a simple polynomial by replacing every term – just increment the exponent first, *then* divide the coefficient by the new exponent each term cx^n with $\dfrac{c}{n+1}x^{n+1}$ (exactly the opposite steps and order from differentiation). This rule only works when "x" is a simple variable.

Remember to *always* add "+ C" at the end of any *indefinite* integral!! (Why? Since differentiation drops constants, without more information we can't restore the constant when we reverse it.)

Find $\int (16x^7+15x^2-4)dx$.   Answer: $2x^8 + 5x^3 – 4x + C$.

Find $\int (12x^5+20x^4-7)dx$.   Answer: $2x^6 + 4x^5 – 7x + C$.

Find $\int 72x^7-21x^6-54x^5 dx$.   Answer: $9x^8 – 3x^7 – 9x^6 + C$.

Find $\int 110x^9+12x^2-18x+8 dx$.   Answer: $11x^{10} + 4x^3 – 9x^2 + 8x + C$.

Definite Integrals

When you want to find a definite integral (that is, the area from x=a to x=b) for some function f(x), you first find the indefinite integral (call it F(x)) and calculate F(b) – F(a). You can ignore the "C" in this case; since C-C is always 0, the unknown constant C will always cancel out. In short:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Find $\int_3^5 (4x)\,dx$.

Answer: Indefinite integral $F(x) = 2x^2 + C$, so
$F(5) - F(3) = (2(5)^2 + C) - (2(3)^2 + C) = 50 - 18 = 32$

Find $\int_5^7 (8x^3)\,dx$.

Answer: Indefinite integral $F(x) = 2x^4 + C$, so
$F(7) - F(5) = (2(7)^4 + C) - (2(5)^4 + C) = 4802 - 1250 = 3552$

Find $\int_3^6 16x^7 + 15x^2 - 4\,dx$.

Answer: Indefinite integral $F(x) = 2x^8 + 5x^3 - 4x + C$, so
$F(6) - F(3) = (2(6)^8 + 5(6)^3 - 4(6) + C) - (2(3)^8 + 5(3)^3 - 4(3) + C) = 3360288 - 13245 = 3347043$